LP Model Formulation

- Decision variables
  - Depends on the type of the LP problem
  - These variables are controllable
  - The variables can be the quantities of the resources to be allocated or the number of units to be produced or sold
  - The decision maker has to determine the value of these variables
- Objective function
  - a linear relationship reflecting the objective of an operation
  - most frequent objective of business firms is to *maximize profit*
  - most frequent objective of individual operational units (such as a production or packaging department) is to *minimize cost*
- Constraint
  - a linear relationship representing a restriction on decision making

LP Problems on Product Mix

- Objective

  To select the mix of products or services that results in maximum profits for the planning period

- Decision Variables

  How much to produce and market of each product or service for the planning period

- Constraints

  Maximum amount of each product or service demanded; Minimum amount of product or service policy will allow; Maximum amount of resources available

LP Problems on Production Plan

- Objective

  To select the mix of products or services that results in maximum profits for the planning period

- Decision Variables

  How much to produce on straight-time labor and overtime labor during each month of the year

- Constraints

  Amount of products demanded in each month; Maximum labor and machine capacity available in each month; Maximum inventory space available in each month

Recognizing LP Problems

Characteristics of LP Problems

- A well-defined single objective must be stated.
- There must be alternative courses of action.
- The total achievement of the objective must be constrained by scarce resources or other restraints.
- The objective and each of the constraints must be expressed as linear mathematical functions.

Steps in Formulating LP Problems

1. Define the objective. (min or max)
2. Define the decision variables. (positive, binary)
3. Write the mathematical function for the objective.
4. Write a 1- or 2-word description of each constraint.
5. Write the right-hand side (RHS) of each constraint.
6. Write $\leq$, =, or $\geq$ for each constraint.
7. Write the decision variables on LHS of each constraint.
8. Write the coefficient for each decision variable in each constraint.

Example: LP Formulation

Cycle Trends is introducing two new lightweight bicycle frames, the Deluxe and the Professional, to be made from aluminum and steel alloys. The anticipated unit profits are $10 for the Deluxe and $15 for the Professional.

The number of pounds of each alloy needed per frame is summarized on the next slide. A supplier delivers 100 pounds of the aluminum alloy and 80 pounds of the steel alloy weekly. How many Deluxe and Professional frames should Cycle Trends produce each week?

Example: LP Formulation

Pounds of each alloy needed per frame

	Aluminum Alloy	Steel Alloy
Deluxe	2	3
Professional	2	4

Example: LP Formulation

Define the objective

Maximize total weekly profit

Define the decision variables

x_1 = number of Deluxe frames produced weekly

x_2 = number of Professional frames produced weekly

Write the mathematical objective function

$$\text{Max } Z = 10x_1 + 15x_2$$

Example: LP Formulation

Write a one- or two-word description of each constraint

Aluminum available

Steel available

Write the right-hand side of each constraint

100

80

Write <, =, > for each constraint

≤ 100

≤ 80

Example: LP Formulation

- Write all the decision variables on the left-hand side of each constraint

$x_1 \quad x_2 \leq 100$

$x_1 \quad x_2 \leq 80$

Write the coefficient for each decision in each constraint

$+ 2x_1 + 2x_2 \leq 100$

$+ 3x_1 + 4x_2 \leq 80$

Example: LP Formulation

- LP in Final Form

  Max $Z = 10x_1 + 15x_2$

  Subject To

  $2x_1 + 2x_2 \leq 100$ (aluminum constraint)

  $3x_1 + 4x_2 \leq 80$ (steel constraint)

  $x_1 , x_2 \geq 0$ (non-negativity constraints)

LP Model Formulation (cont.)

Max/min $z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

subject to:

$$\left\{\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \ (\leq, =, \geq)\ b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \ (\leq, =, \geq)\ b_2 \\ \quad : \\ a_{m1}x1 + a_{m2}x_2 + \ldots + a_{mn}x_n \ (\leq, =, \geq)\ b_m \end{array}\right.$$

x_j = decision variables

b_i = constraint levels

c_j = objective function coefficients

a_{ij} = constraint coefficients

LP Model: Example

	RESOURCE REQUIREMENTS		
PRODUCT	*Labor (hr/unit)*	*Clay (lb/unit)*	*Revenue ($/unit)*
Bowl	1	4	40
Mug	2	3	50

There are 40 hours of labor and 120 pounds of clay available each day

Decision variables

x_1 = number of bowls to produce

x_2 = number of mugs to produce

LP Formulation: Example

Maximize $Z = 40\ x_1 + 50\ x_2$

Subject to

$$x_1 + 2x_2 \leq 40 \text{ hr} \quad \text{(labor constraint)}$$
$$4x_1 + 3x_2 \leq 120 \text{ lb} \quad \text{(clay constraint)}$$
$$x_1, x_2 \geq 0$$

Solution is $x_1 = 24$ bowls $x_2 = 8$ mugs

Revenue = 1,360

Example: LP Formulation

Montana Wood Products manufacturers two-high quality products, tables and chairs. Its profit is Rs15 per chair and Rs 21 per table. Weekly production is constrained by available labor and wood. Each chair requires 4 labor hours and 8 board feet of wood while each table requires 3 labor hours and 12 board feet of wood. Available wood is 2400 board feet and available labor is 920 hours. Management also requires at least 40 tables and at least 4 chairs be produced for every table produced. To maximize profits, how many chairs and tables should be produced?

Example: LP Formulation

Define the objective

- Maximize total weekly profit

Define the decision variables

- x_1 = number of chairs produced weekly
- x_2 = number of tables produced weekly

Write the mathematical objective function

- Max $Z = 15x_1 + 21x_2$

Example: LP Formulation

Write a one- or two-word description of each constraint

- Labor hours available
- Board feet available
- At least 40 tables
- At least 4 chairs for every table

Write the right-hand side of each constraint

- 920
- 2400
- 40
- 4 to 1 ratio

Write <, =, > for each constraint

- $\leq$ 920
- $\leq$ 2400
- $\geq$ 40
- 4 to 1

Example: LP Formulation

- Write all the decision variables on the left-hand side of each constraint
  - $x_1 \quad x_2 \leq 920$
  - $x_1 \quad x_2 \leq 2400$
  - $x_2 \geq 40$
  - 4 to 1 ratio

Write the coefficient for each decision in each constraint

  - $+ 4x_1 + 3x_2 \leq 920$
  - $+ 8x_1 + 12x_2 \leq 2400$
  - $x_2 \geq 40$
  - $x_1 \geq 4 x_2$

Example: LP Formulation

- LP in Final Form
  - Max $Z = 15x_1 + 21x_2$
  - Subject To
    - $4x_1 + 3x_2 \leq 920$ (labor constraint)
    - $8x_1 + 12x_2 \leq 2400$ (wood constraint)
    - $x_2 \geq 40$ (make at least 40 tables)
    - $x_1 \geq 4$ (at least 4 chairs for every table)
    - $x_1 , x_2 \geq 0$ (non-negativity constraints)

Example: LP Formulation

The Sureset Concrete Company produces concrete. Two ingredients in concrete are sand (costs Rs 6 per ton) and gravel (costs Rs 8 per ton). Sand and gravel together must make up exactly 75% of the weight of the concrete. Also, no more than 40% of the concrete can be sand and at least 30% of the concrete be gravel. Each day 2000 tons of concrete are produced. To minimize costs, how many tons of gravel and sand should be purchased each day?

Example: LP Formulation

Define the objective

- Minimize daily costs

Define the decision variables

- x_1 = tons of sand purchased
- x_2 = tons of gravel purchased

Write the mathematical objective function

- Min $Z = 6x_1 + 8x_2$

Example: LP Formulation

Write a one- or two-word description of each constraint

- 75% must be sand and gravel
- No more than 40% must be sand
- At least 30% must be gravel

Write the right-hand side of each constraint

- .75(2000)
- .40(2000)
- .30(2000)

Write <, =, > for each constraint

- = 1500
- $\leq$ 800
- $\geq$ 600

Example: LP Formulation

- <u>Write all the decision variables on the left-hand side of each constraint</u>
  - $x_1 \quad x_2 \;= 1500$
  - $x_1 \leq 800$
  - $x_2 \geq 600$

<u>Write the coefficient for each decision in each constraint</u>

  - $+ x_1 + x_2 \;= 1500$
  - $+ x_1 \leq 800$
  - $x_2 \geq 600$

Example: LP Formulation

- <u>LP in Final Form</u>
  - Min $Z = 6x_1 + 8x_2$
  - Subject To
    - $x_1 + x_2 = 1500$ (mix constraint)
    - $x_1 \leq 800$ (mix constraint)
    - $x_2 \geq 600$ (mix constraint)
    - $x_1 , x_2 \geq 0$ (non-negativity constraints)

LP Problem

- Galaxy Ind. produces two water guns, the Space Ray and the Zapper. Galaxy earns a profit of Rs3 for every Space Ray and Rs2 for every Zapper. Space Rays and Zappers require 2 and 4 production minutes per unit, respectively. Also, Space Rays and Zappers require .5 and .3 pounds of plastic, respectively. Given constraints of 40 production hours, 1200 pounds of plastic, Space Ray production can't exceed Zapper production by more than 450 units; formulate the problem such that Galaxy maximizes profit.

LP Model

R = # of Space Rays to produce

Z = # of Zappers to produce

Max Z = 3.00R + 2.00Z

ST

2R + 4Z ≤ 2400   can't exceed available hours (40*60)

.5R + .3Z ≤ 1200   can't exceed available plastic

R - S ≤ 450   Space Rays can't exceed Zappers by more than 450

R, S ≥ 0   non-negativity constraint

Formulating LPP

- https://www.youtube.com/watch?v=oaEmCiKMmdI