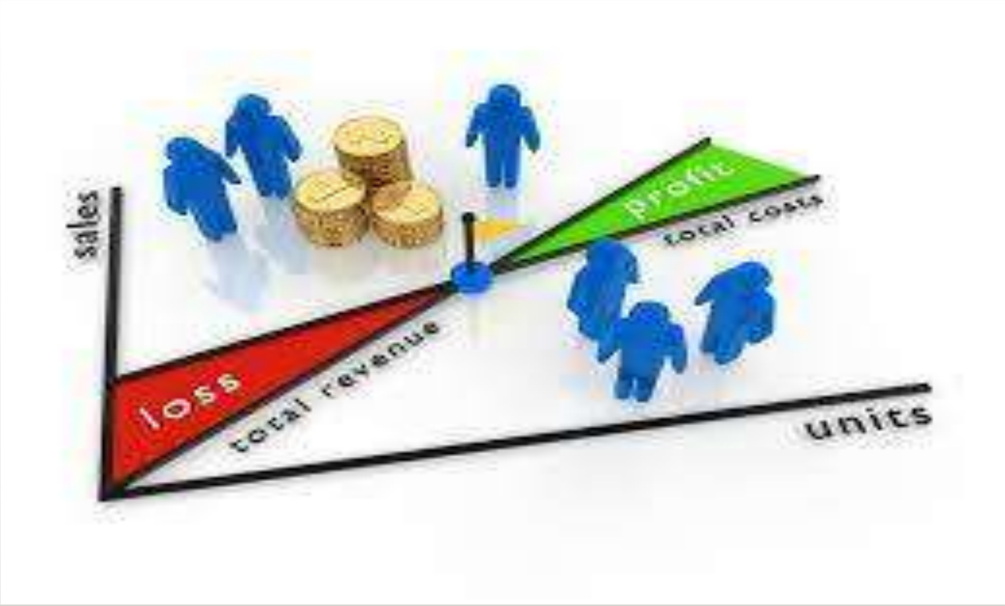
Linear programming PROBLEM



What is LP ?

The word linear means the relationship which can be represented by a straight line .i.e the relation is of the form
$ax + by = c$.

- In other words it is used to describe the relationship between two or more variables which are proportional to each other
- The word “programming” is concerned with the optimal allocation of limited resources.
- Linear programming is a way to handle certain types of optimization problems

Linear Programming

- LP is a mathematical modeling technique useful for the allocation of “scarce or limited” resources such as labor, material, machine ,time ,warehouse space ,etc…,to several competing activities such as product ,service ,job, new equipments, projects, etc...on the basis of a given criteria of optimality

Definition of LP

- A mathematical technique used to obtain an optimum solution in resource allocation problems, such as production planning.
- It is a mathematical model or technique for efficient and effective utilization of limited recourses to achieve organization objectives (Maximize profits or Minimize cost).
- When solving a problem using linear programming , the program is put into a number of linear inequalities and then an attempt is made to maximize (or minimize) the inputs

Definition of LPP

- There must be well defined objective function.
- There must be a constraint on the amount.
- There must be alternative course of action.
- The decision variables should be interrelated and non negative.
- The resource must be limited in supply.

Requirements

- Proportionality
- Additivity
- Continuity
- Certainity
- Finite Choices

Assumptions

- Business
- Industrial
- Military
- Economic
- Marketing
- Distribution

Application Of linear Programming

- Industrial Application
  - Product Mix Problem
  - Blending Problems
  - Production Scheduling Problem
  - Assembly Line Balancing
  - Make-Or-Buy Problems
- Management Applications
  - Media Selection Problems
  - Portfolio Selection Problems
  - Profit Planning Problems
  - Transportation Problems
- Miscellaneous Applications
  - Diet Problems
  - Agriculture Problems
  - Flight Scheduling Problems
  - Facilities Location Problems

Areas of application of Linear programming

- It helps in attaining optimum use of productive factors.
- It improves the quality of the decisions.
- It provides better tools for meeting the changing conditions.
- It highlights the bottleneck in the production process.

Advantages of L.P.

- For large problems the computational difficulties are enormous.
- It may yield fractional value answers to decision variables.
- It is applicable to only static situation.
- LP deals with the problems with single objective.

Limitation of L.P.

- Graphical Method
- Simplex Method

Types of Solutions to L.P. Problem

- The canonical form
  - Objective function is of maximum type
  - All decision variables are non negative

- The Standard Form
  - All variables are non negative
  - The right hand side of each constraint is non negative.
  - All constraints are expressed in equations.
  - Objective function may be of maximization or minimization type.

Forms of L.P.

Mathematical model of LPP

Optimize (Maximize or Minimize) the objective function:

$Z = C_1X_1 + C_2X_2 + \dots + C_nX_n$

Subject to satisfaction of m- constraints:

$$\left.\boxed{\begin{array}{c} a_{11} X_1 + a_{12} X_2 + \dots + a_{1n} X_n \ (\leq = \geq)\ b_1 \\ a_{21} X_1 + a_{22} X_2 + \dots + a_{2n} X_n \ (\leq = \geq)\ b_2 \\ a_{i1} X_1 + a_{i2} X_2 + \dots + a_{in} X_n \ (\leq = \geq)\ b_i \\ a_{m1} X_1 + a_{m2} X_2 + \dots + a_{mn} X_n \ (\leq = \geq)\ b_m \end{array}}\right\}$$

Where the constraint may be in the form of an inequality (≤ or ≥) or even in the form of an equality (=) and finally satisfy the non-negativity restrictions

$X_1, X_2, X_n \geq 0$

Mathematical model of LPP (Cont.)

where C_j (j=1, 2.,n); b_i (i=1, 2,…m) and a_{ij} are all constants and m<n,

decision variables $X_j \geq 0$, j=1, 2,…,n.

If b_i is the available amount of resource i then a_{ij} is amount of resource i that must be allocated (technical coefficient) to each unit of activity j.

NOTE: By convention, the values of RHS parameters b_i (i=1, 2, 3,…,m) are restricted to non-negative values only. If any value of b_i is negative then it is to be changed to a positive value by multiplying both sides of the constraint by -1. This not only changes the sign of all LHS Coefficients and of RHS parameters but also changes the direction of inequality sign.

Solution:

A set of variables [X1,X2,...,Xn+m] is called a solution to L.P. Problem if it satisfies its constraints.

Feasible Solution:

A set of variables [X1,X2,...,Xn+m] is called a feasible solution to L.P. Problem if it satisfies its constraints as well as non-negativity restrictions.

Optimal Feasible Solution:

The basic feasible solution that optimises the objective function.

Unbounded Solution:

If the value of the objective function can be increased or decreased indefinitely, the solution is called an unbounded solution.

Important definitions in L.P.

- Slack Variable
- Surplus Variable
- Artificial Variable

Variables used in L.P.

Non-negative variables, Subtracted from the L.H.S of the constraints to change the inequalities to equalities. Added when the inequalities are of the type (>=). Also called as “negative slack”.

- Slack Variables

Non-negative variables, added to the L.H.S of the constraints to change the inequalities to equalities. Added when the inequalities are of the type (<=).

- Surplus Variables

In some L.P problems slack variables cannot provide a solution. These problems are of the types (>=) or (=) . Artificial variables are introduced in these problems to provide a solution.

- Artificial Variables

Artificial variables are fictitious and have no physical meaning.

- For every L.P. problem there is a related unique L.P. problem involving same data which also describes the original problem.
- The primal programme is rewritten by transposing the rows and columns of the algebraic statement of the problem.
- The variables of the dual programme are known as “Dual variables or Shadow prices” of the various resources.
- The optimal solution of the dual problem gives complete information about the optimal solution of the primal problem and vice versa.

DUALITY :

- By converting a primal problem into dual , computation becomes easier , as the no. of rows(constraints) reduces in comparison with the no. of columns(variables).
- Gives additional information as to how the optimal solution changes as a result of the changes in the coefficients . This is the basis for sensitivity analysis.
- Economic interpretation of dual helps the management in making future decisions.
- Duality is used to solve L.P. problems in which the initial solution in infeasible.

ADVANTAGES :

Two situations:

- In formulation , it is assumed that the parameters such as market demand, equipment capacity, resource consumption, costs, profits etc., do not change but in real time it is not possible.

- After attaining the optimal solution, one may discover that a wrong value of a cost coefficient was used or a particular variable or constraint was omitted etc.,

SENSITIVITY ANALYSIS : (Post Optimality test)

- Changes in the parameters of the problem may be discrete or continuous.
- The study of effect of discrete changes in parameters on the optimal solution is called as “Sensitivity analysis”.
- The study of effect of continuous changes in parameters on the optimal solution is called as “Parametric Programming.”
- The objective of the sensitivity analysis is to determine how sensitive is the optimal solution to the changes in the parameters.