Transportation Model

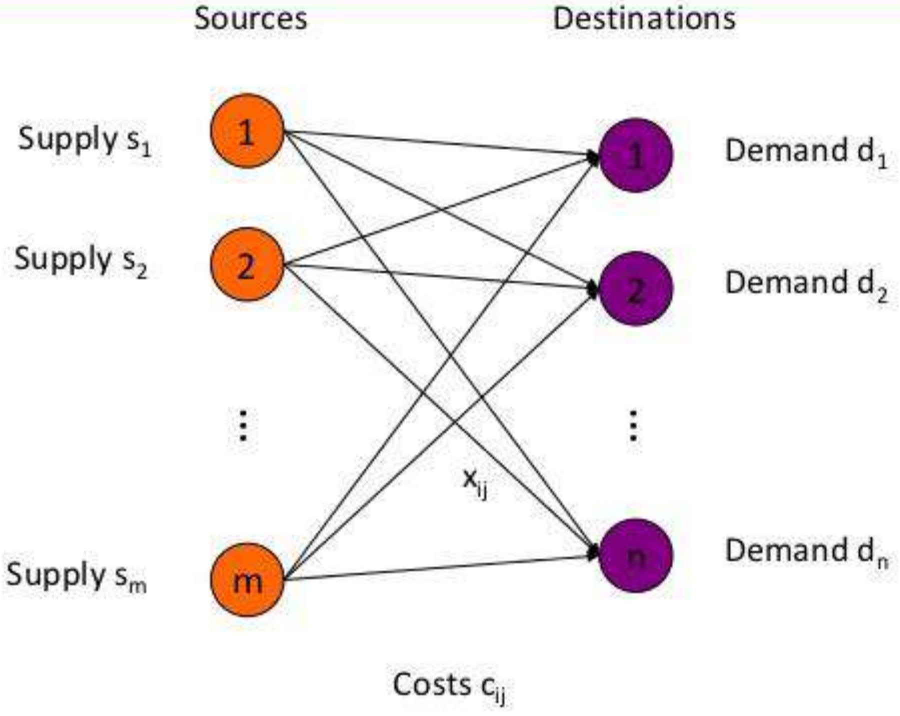
The Transportation Model

The transportation model is a special class of LPPs that deals with transporting(=shipping) a commodity from **sources** (e.g. factories) to **destinations** (e.g. warehouses).

The objective is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits.

We assume that the shipping cost is proportional to the number of units shipped on a given route.

Simple Network Representation



Application of Transportation Problem

- ❑ Minimize shipping costs
- ❑ Determine low cost location
- ❑ Find minimum cost production schedule
- ❑ Military distribution system

Transportation Problem

- How much should be shipped from several sources to several destinations
  - Sources: Factories, warehouses, etc.
  - Destinations: Warehouses, stores, etc.
- Transportation models
  - Find lowest cost shipping arrangement
  - Used primarily for existing distribution systems

A Transportation Model Requires

- The origin points, and the capacity or supply per period at each
- The destination points and the demand per period at each
- The cost of shipping one unit from each origin to each destination

Transportation problem

Warehouse

		1	2	3	4	Supply	
	A	$2	$4	$4	$1	150	
Factory	B	10	3	7	7	200	
	C	6	7	20	5	150	
Demand		50	100	150	200	500	500

We assume that there are m sources 1,2, …, m and n destinations 1, 2, …, n. The cost of shipping one unit from Source i to Destination j is c_{ij}.

We assume that the availability at source i is a_i (i=1, 2, …, m) and the demand at the destination j is b_j (j=1, 2, …, n). We make an important assumption: the problem is a **balanced** one. That is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

That is, total availability equals total demand.

TP Model

Destinations

		1	2	3	… j …	n	Supply
	1	C_{11} x_{11}	C_{12} x_{12}	C_{13} x_{13}	C_{1j} x_{1j}	C_{1n} x_{1n}	a_1
	2	C_{21} x_{21}	C_{22} x_{22}	C_{23} x_{23}	C_{2j} x_{2j}	C_{2n} x_{2n}	a_2
Sources or Origins	3	C_{31} x_{31}	C_{32} x_{32}	C_{33} x_{33}	C_{3j} x_{3j}	C_{3n} x_{3n}	a_3
	:	C_{i1} x_{i1}	C_{i2} x_{i2}	C_{i3} x_{i3}	C_{ij} x_{ij}	C_{in} x_{in}	: a_i :
	m	C_{m1} x_{m1}	C_{m2} x_{m2}	C_{m3} x_{m3}	C_{mj} x_{mj}	C_{mn} x_{mn}	a_m
Demand		b_1	b_2	b_3	…b_j…	b_n	

TP Model

- In the table , c_{ij} , i = 1,2, ….., m ; j = 1,2, …… , n , is the unit shipping cost from the ith origin to jth destination
- x_{ij} is the quantity shipped from the ith origin to jth destination
- a_i is the supply available at origin i
- b_j is the demand at destination j

Two Types of Transportation Problem

- Balanced Transportation Problem
  where the total supply equals total demand

- Unbalanced Transportation Problem
  where the total supply is not equal to the total demand

Balancing an Unbalanced TP

- Introduce a dummy source (if the total demand is more than the total supply)
- Introduce a dummy destination (if the total supply is more than the total demand)
- Cost of shipping in dummy row or column is taken as 0

A few terms used in connection with transportation models

1. **Feasible solution:** A feasible solution to a transportation problem is a set of non-negative allocations, x_{ij} that satisfies the rim (row and column) restrictions.

2. **Basic feasible solution:** A feasible solution to a transportation problem is said to be a basic feasible solution if it contains no more than m + n – 1 non – negative allocations, where m is the number of rows and n is the number of columns of the transportation problem.

A few terms used in connection with transportation models

3. Optimal solution: A feasible solution (not necessarily basic) that minimizes (maximizes) the transportation cost (profit) is called an optimal solution.

4. **Non -degenerate basic feasible solution:** A basic feasible solution to a (m x n) transportation problem is said to be non – degenerate if,

the total number of non-negative allocations is exactly m + n – 1 (i.e., number of independent constraint equations), and

these m + n – 1 allocations are in independent positions.

A few terms used in connection with transportation models

5. Degenerate basic feasible solution: A basic feasible solution in which the total number of non-negative allocations is less than m + n – 1 is called degenerate basic feasible solution.

Steps involved in solution of transportation problem

- To find an initial basic feasible solution (IBFS)
- To check the above solution for optimality
- To revise the solution