U-V method/ Modified Distribution Method/ Modi Method

Checking for optimality after initial solution has been obtained

Recalling the steps in solving TP

- To find an initial basic feasible solution (IBFS)
- To check the above solution for optimality
- To revise the solution

IBFS for a given TP;

- TC = (200 * 3) + (50 * 1) +

  (250 * 6) + (100 * 5) +

  (250 * 3) + (150 * 2)

  = 3700
- Now we have to check for optimality i.e., the TC of 3700 is optimum or can it be reduced further?

200 3	50 1	7	4
2	250 6	100 5	9
8	3	250 3	150 2

IBFS

- Cells in which allocations are made are called occupied cells or basic cells or allocated cells
- Cells in which no allocations are made are called non-basic cells
- When checking for optimality we have to evaluate non basic cells to check if allocating into these cells will reduce the total cost.

Condition for applying optimality test

- Check whether **m + n – 1 is equal to the total number of allocated cells** or not where **m** is the total number of rows and **n** is the total number of columns.

- In this case m = 3, n = 4 and total number of allocated cells is 6 so m + n – 1 = 6.

- (The case when m + n – 1 is not equal to the total number of allocated cells is a case of degeneracy)

Modi method of optimality testing

- For U-V method the values $\mathbf{u_i}$ and $\mathbf{v_j}$ have to be found for the rows and the columns respectively.
- As there are three rows so three $\mathbf{u_i}$ values have to be found i.e. $\mathbf{u_1}$ for the first row, $\mathbf{u_2}$ for the second row and $\mathbf{u_3}$ for the third row.
- Similarly, for four columns four $\mathbf{v_j}$ values have to be found i.e. $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$ and $\mathbf{v_4}$.

How to find out the value of $D_{ij} = C_{ij} - (u_i + v_j)$?

1. To find the values of u_i and v_j using the formula $u + v = c$
2. To find $u_i + v_j$ for empty cells
3. To find $D_{ij} = C_{ij} - (u_i + v_j)$
4. Where c is the original cost given in the problem

U-v method/ modi method

Occupied cells – c_{11}, c_{12}, c_{22}, c_{23}, c_{33}, c_{34}

	V1=	V2=	V3=	V4=
u1 =	200 3	50 1	7	4
u2 =	2	250 6	100 5	9
u3 =	8	3	250 3	150 2

Finding u_i and v_j values for basic cells

- $\mathbf{u_i + v_j = C_{ij}}$ where $\mathbf{C_{ij}}$ is the cost value (only for the allocated cells)
- Start by assigning any of the three $\mathbf{u_i}$ or any of the four $\mathbf{v_j}$ **values** as 0
- Let us assign $\mathbf{u_1 = 0}$ in this case
- Then using the above formula we will get $\mathbf{v_1 = 3}$ as $\mathbf{u_1 + v_1 = 3}$ (i.e. $\mathbf{C_{11}}$)
- $\mathbf{v_2 = 1}$ as $\mathbf{u_1 + v_2 = 1}$ (i.e. $\mathbf{C_{12}}$)
- Similarly, we have got the value for $\mathbf{v_2 = 3}$ so we get the value for $\mathbf{u_2 = 5}$ which implies $\mathbf{v_3 = 0}$.
- From the value of $\mathbf{v_3 = 0}$ we get $\mathbf{u_3 = 3}$ which implies $\mathbf{v_4 = -1}$

u_i and v_j values

	$v_1=3$	$v_2=1$	$v_3=0$	$v_4=-1$
$u_1=0$	200 / 3	250 / 1	7	4
$u_2=5$	2	50 / 6	100 / 5	9
$u_3=3$	8	3	250 / 3	150 / 2

Net evaluations for unallocated cells

- **d_{ij} = C_{ij} - [u_i + v_j] for each unoccupied cell i.e., cells in which no allocation is made earlier**

1. For **C_{13}**, **d_{13} = 7 – [0 + 0] = 7** (here **C_{13} = 7**, **u_1 = 0** and **v_3 = 0**)
2. For **C_{14}**, **d_{14} = 4 – [0 + (-1)] = 5**
3. For **C_{21}**, **d_{21} = 2 – [5 + 3] = -6**
4. For **C_{24}**, **d_{24} = 9 – [5 + (-1)] = 5**
5. For **C_{31}**, **d_{31} = 8 – [3 + 3] = 2**
6. For **C_{32}**, **d_{32} = 3 – [3 + 1] = -1**

Optimality rule: stop if all ($\mathbf{d_{ij}} \geq 0$)

- If all net evaluations $\mathbf{d_{ij}}$ **are zero or positive, then the total cost cannot be reduced further;**
- **Current total cost is the optimal total cost and the current solution is the optimal solution;**
- **Existence of negative d_{ij} s implies scope for improving the solution;**
- **Choose the cell having most negative d_{ij} value to enter the basis;**
- **Here most negative value is -6 and corresponds to cell C_{21}**
- Now this cell is new basic cell. This cell will also be included in the solution.

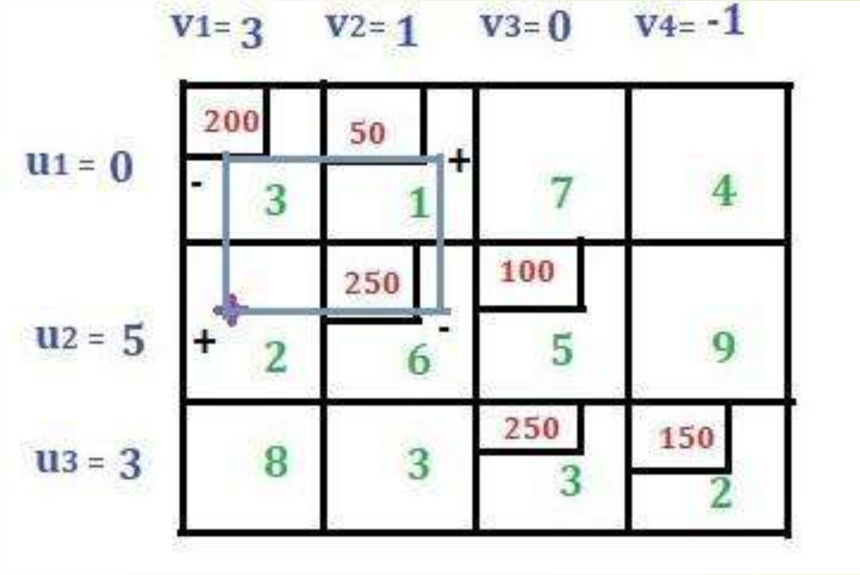
Moving towards optimality

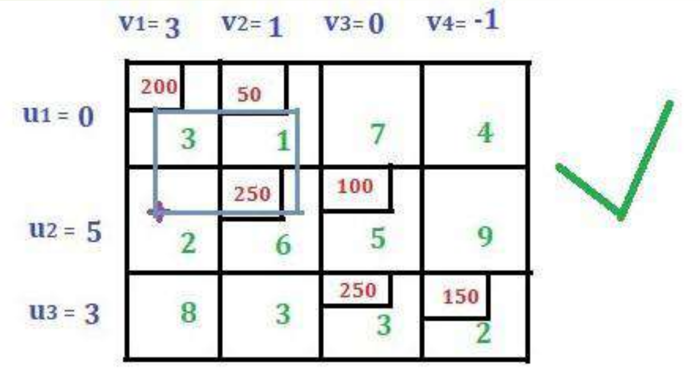
- Form loop from the chosen non-basic cell
- Starting from the new basic cell draw a closed-path in such a way that the right angle turn is done only at the allocated cell or at the new basic cell

	v1= 3	v2= 1	v3= 0	v4= -1
u1 = 0	200 3	50 1	7	4
u2 = 5	+ 2	250 6	100 5	9
u3 = 3	8	3	250 3	150 2

Moving towards optimality

- Assign alternate plus-minus sign to all the cells with right angle turn (or the corner) in the loop with plus sign assigned at the new basic cell

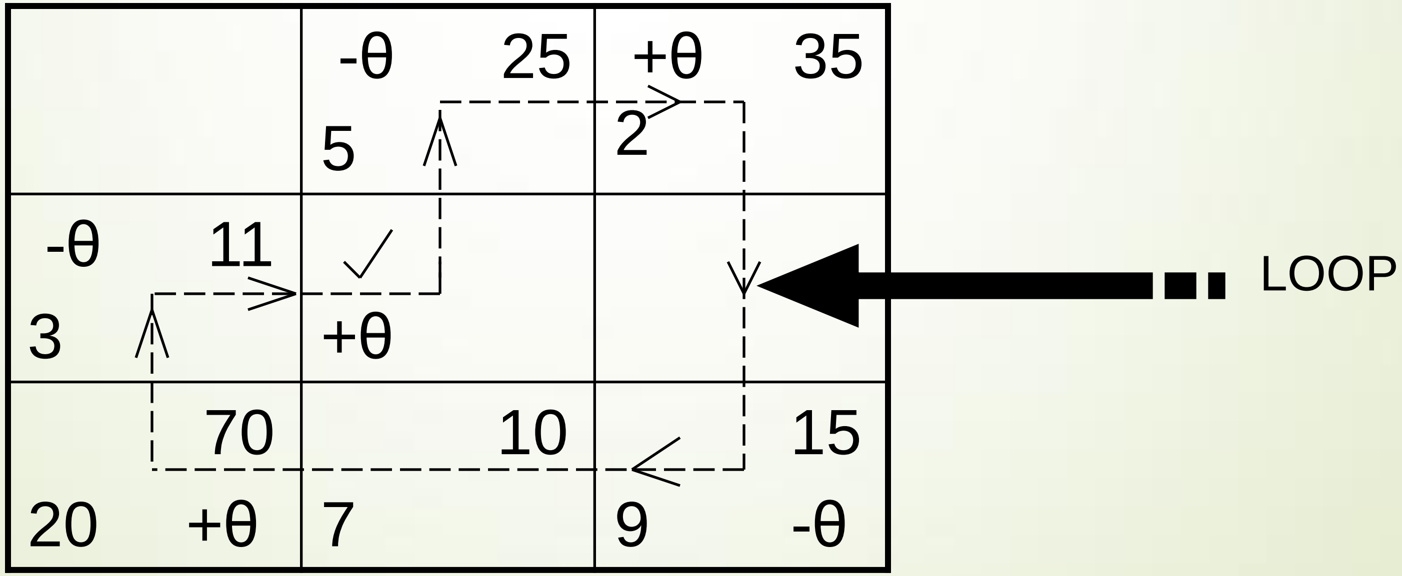




How to revise the solution?

- Mark +θ in the place where there is a negative value
- Proceed with the loop
- Direction of loop can be changed at only places where there is a allotment
- mark + θ and – θ where the loop changes its direction
- Observe – θ cells and take the least allocation
- Add the value of θ where + θ is there and subtract the value of θ where – θ is there

Model of a loop



Revising the allocations

- Consider the cells with a negative sign. Compare the allocated value (i.e. 200 and 250 in this case) and select the minimum (i.e. select 200 in this case)
- Now subtract 200 from the cells with a minus sign and add 200 to the cells with a plus sign
- Draw a new iteration
- Cell C_{11} goes away from the basis and cell C_{21} becomes the new basic cell

Revised allocations and the new solution

3	250 / 1	7	4
200 / 2	50 / 6	100 / 5	9
8	3	250 / 3	150 / 2

- Revised TC :

  (250 * 1) + (200*2) +

  (50 * 6) + (100 * 5) +

  (250 * 3) + (150 * 2)

  = 2500

- Note that allocations will change only in cells with + or – sign. All other allocations remain the same

From initial to improved solution

Initial solution and initial TC = 3700

200 (3)	50 (1)	7	4
2	250 (6)	100 (5)	9
8	3	250 (3)	150 (2)

Revised solution and revised TC = 2500

3	250 (1)	7	4
200 (2)	50 (6)	100 (5)	9
8	3	250 (3)	150 (2)

Optimality testing

- Test the revised solution for optimality. Stop if all net evaluations are zero or positive.
- Check the total number of allocated cells is equal to (m + n – 1)
- Again find u_i values and v_j values using the formula $\mathbf{u_i + v_j = C_{ij}}$ where $\mathbf{C_{ij}}$ is the cost value only for allocated cell
- Assign $\mathbf{u_1 = 0}$ then we get $\mathbf{v_2 = 1}$. Similarly, we will get following values for $\mathbf{u_i}$ and $\mathbf{v_j}$

u_i and v_j values and net evaluations $\mathbf{d_{ij}}$

	$v_1=-3$	$v_2=1$	$v_3=0$	$v_4=-1$
$u_1=0$	3	250 1	7	4
$u_2=5$	200 2	50 6	100 5	9
$u_3=3$	8	3	250 3	150 2

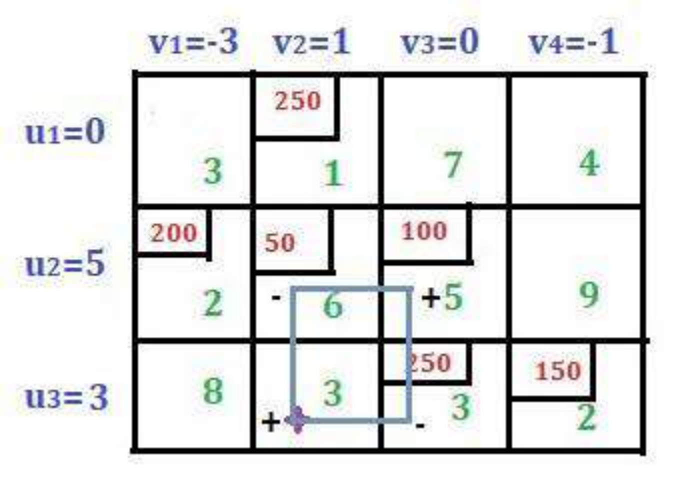
1. For C_{11}, d_{11} = 3 – [0 + -3] = 6
2. For C_{13}, d_{13} = 7 – [0 + 0] = 7
3. **For C_{14}, d_{14} = 4 – [0 + (-1)] = 5**
4. For C_{24}, d_{24} = 9 – [5 + (-1)] = 5
5. For C_{31}, d_{31} = 8 – [3+-3] = 8
6. For C_{32}, d_{32} = 3 – [3+ 1] = -1

Optimality rule: stop if all ($d_{ij} \geq 0$)

- If all net evaluations d_{ij} **are zero or positive, then the total cost cannot be reduced further;**
- **Current total cost is the optimal total cost and the current solution is the optimal solution;**
- **Existence of negative d_{ij} s implies scope for improving the solution;**
- **Choose the cell having most negative d_{ij} value to enter the basis;**
- **Here most negative value is -1 and corresponds to cell C_{31}**
- Now this cell is new basic cell. This cell will also be included in the solution.

Moving towards optimality

- Form loop from the chosen non-basic cell
- Starting from the new basic cell draw a closed-path in such a way that the right angle turn is done only at the allocated cell or at the new basic cell



Revising the allocations

- Consider the cells with a negative sign. Compare the allocated value (i.e. 50 and 250 in this case) and select the minimum (i.e. select 50 in this case)
- Now subtract 50 from the cells with a minus sign and add 50 to the cells with a plus sign
- Draw a new iteration
- Cell C_{22} goes away from the basis and cell C_{32} becomes the new basic cell

Revised allocations and the new solution

3	250 1	7	4
200 2	6	150 5	9
8	50 3	200 3	150 2

- Revised TC :

  (250 * 1) + (200*2) +

  (150 * 5) + (50 * 3) +

  (200 * 3) + (150 * 2)

  = 2450

- Note that allocations will change only in cells with + or – sign. All other allocations remain the same

From previous solution to improved solution

Initial solution and initial TC = 2500

3	250 / 1	7	4
200 / 2	50 / 6	100 / 5	9
8	3	250 / 3	150 / 2

Revised solution and revised TC = 2450

3	250 / 1	7	4
200 / 2	6	150 / 5	9
8	50 / 3	200 / 3	150 / 2

Optimality testing

- Test the revised solution for optimality. Stop if all net evaluations are zero or positive.
- Check the total number of allocated cells is equal to (m + n – 1)
- Again find u_i values and v_j values using the formula $\mathbf{u_i + v_j = C_{ij}}$ where $\mathbf{C_{ij}}$ is the cost value only for allocated cell
- Assign $\mathbf{u_1}$ = 0 then we get $\mathbf{v_2 = 1}$. Similarly, we will get following values for $\mathbf{u_i}$ and $\mathbf{v_j}$

u_i and v_j values and net evaluations $\mathbf{d_{ij}}$

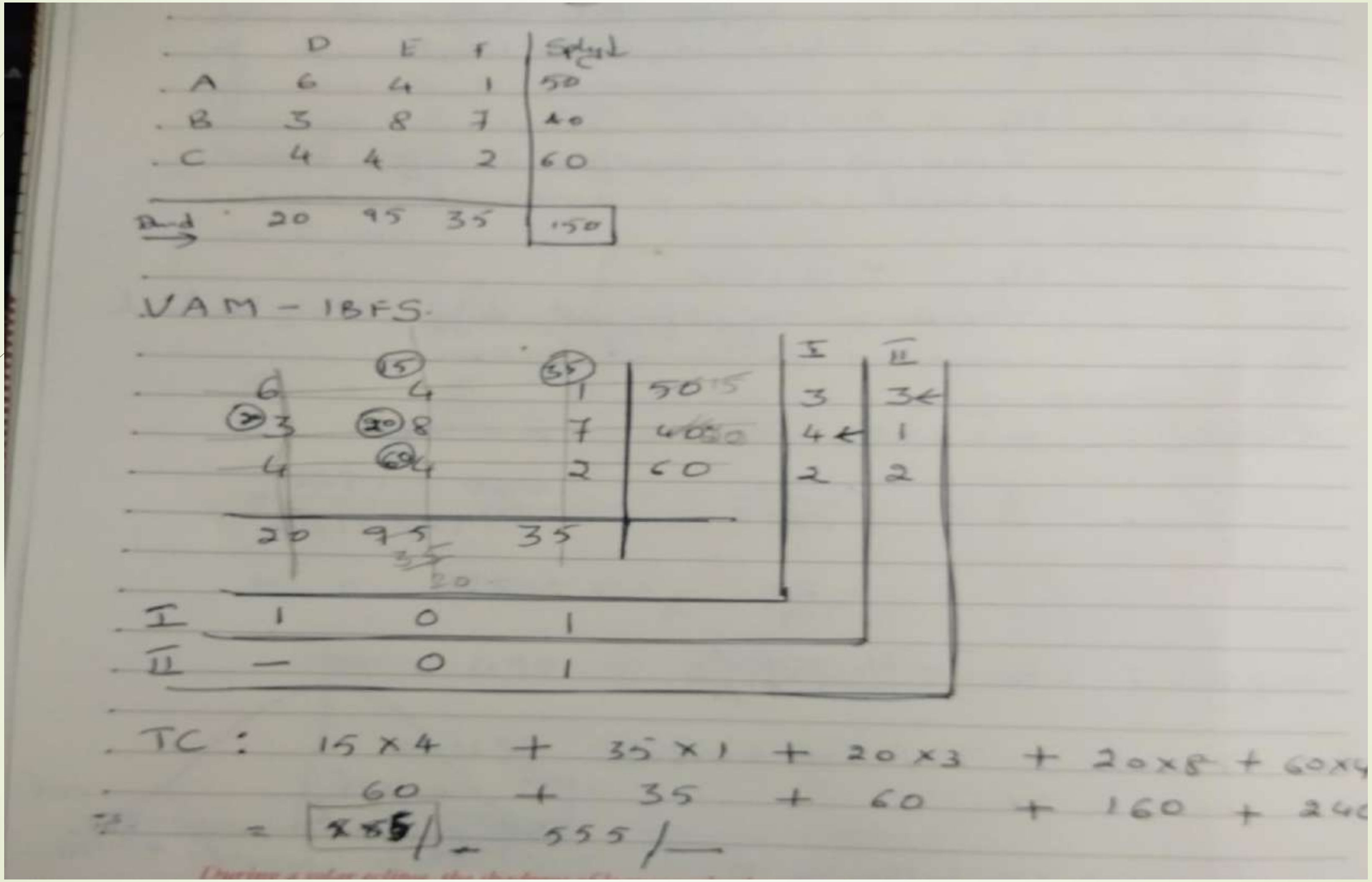
	$v_1=-2$	$v_2=1$	$v_3=1$	$v_4=0$
$u_1=0$	3	250 / 1	7	4
$u_2=4$	200 / 2	6	150 / 5	9
$u_3=2$	8	50 / 3	200 / 3	150 / 2

3

1. For C_{11}, d_{11} = 3 – [0 + -2] = 5
2. For C_{13}, d_{13} = 7 – [0 + 1] = 6
3. **For C_{14}, d_{14} = 4 – [0 + 0] = 4**
4. For C_{22}, d_{24} = 6 – [4+ 1] = 1
5. For C_{24}, d_{24} = 9 – [4 + 0] = 5
6. For C_{31}, d_{31} = 8 – [2+ -2] = 8

Since all net evaluations are positive this is the optimal solution;

Problem 2:



	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

VAM – IBFS

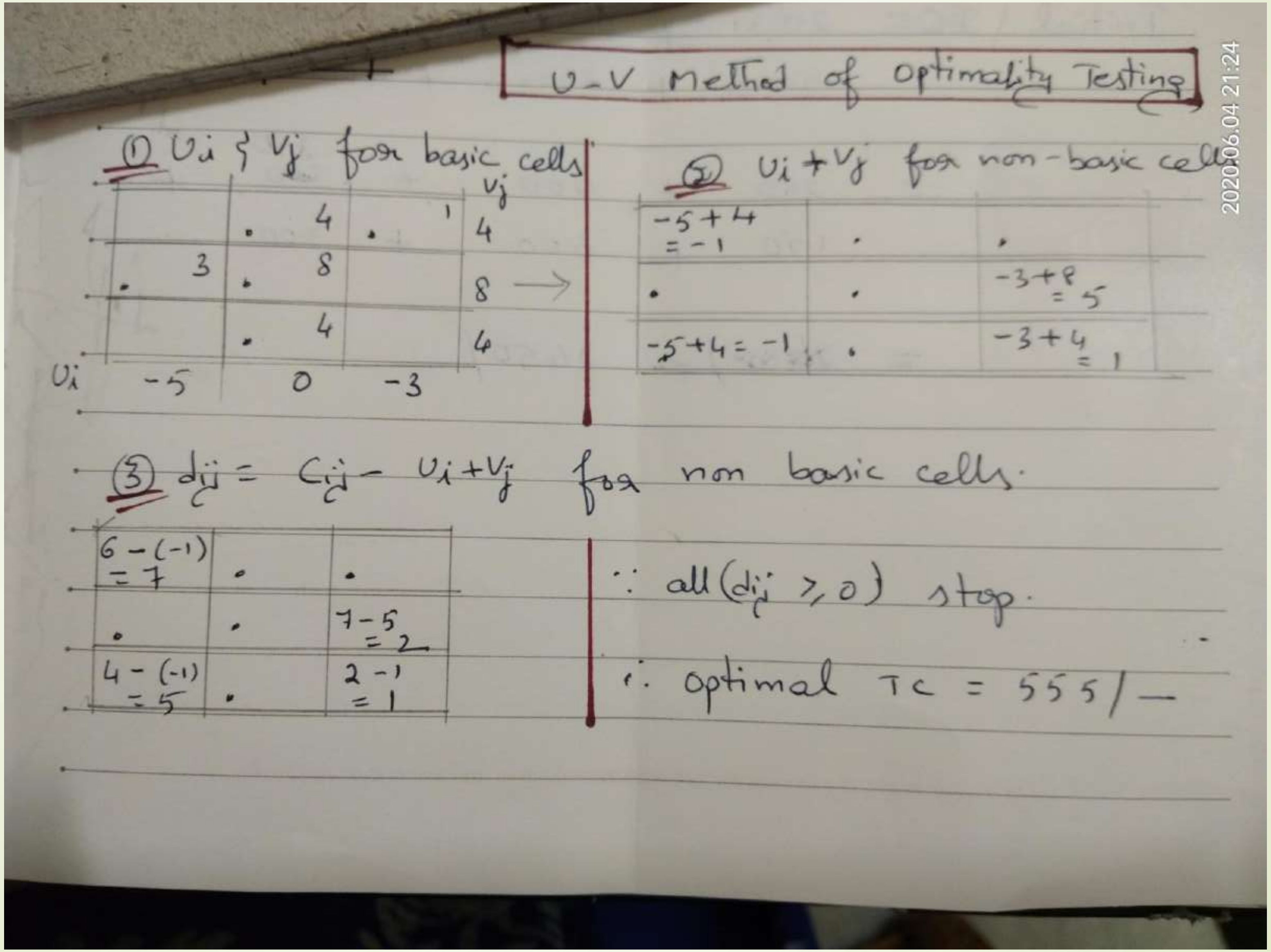
				I	II
6	(15) 4	(35) 1	50 15	3	3 ←
(20) 3	(20) 8	7	40 20	4 ←	1
4	(60) 4	2	60	2	2
20	95 35 20	35			
I: 1	0	1			
II: —	0	1			

TC : 15 × 4 + 35 × 1 + 20 × 3 + 20 × 8 + 60 × 4

= 60 + 35 + 60 + 160 + 240

= 555/-   555/-

Optimality test



U-V Method of optimality Testing

① u_i & v_j for basic cells

			v_j
	4	1	4
3	8		8
	4		4
u_i: -5	0	-3	

② $u_i + v_j$ for non-basic cells

-5+4 = -1	·	·
·	·	-3+8 = 5
-5+4 = -1	·	-3+4 = 1

③ $d_{ij} = c_{ij} - u_i + v_j$ for non basic cells.

6-(-1) = 7	·	·
·	·	7-5 = 2
4-(-1) = 5	·	2-1 = 1

∵ all ($d_{ij} \geq 0$) stop.

∴ optimal TC = 555/-

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